Dynamic Optimal Approach for an Electric Taxi Fleet's Charging and Order-service Schemes

Kaize Yu, Pengyu Yan¹ and Zhibin Chen²
University of Electronic Science and Technology, Chengdu, China
NYU Shanghai, Shanghai, China
Corresponding author: Pengyu Yan, yanpy@uestc.edu.cn

Abstract: This paper addresses an optimal charging and order service integration decision problem for an electric taxi (ET) fleet, operating by an e-platform. Compared with the conventional fuel taxis, the relatively long out-of-service time for recharging ETs significantly affects the revenue of drivers and the service level of the e-platform. In this study, we propose a charging-and-order-serve decision (COSD) system to jointly determine the ETs’ dispatching and charging schemes. The dynamic optimization problem is formulated as a centralized multi-period stochastic model to maximize the total revenue of the fleet over a finite operational horizon. We develop an efficient algorithm based on the framework of rolling-horizon and considering uncertain orders of passengers. The result of the numerical experiment demonstrates the effectiveness of the proposed approach.

Keywords: Electric taxi; charging and order serving; multi-period stochastic model; dynamic decision; rolling horizon;

1 Introduction

The electric vehicle (EV) has become an effective way to alleviate the environmental pollution caused by traditional fuel vehicles. Governments around the world are vigorously promoting electric vehicles. At the same time, a large number of electric vehicles are in operation on the online e-hailing platform. For example, an online hailing platform, Caocao has replaced all its vehicles with electric vehicles. Didi also plans to launch one million electric vehicles to provide travel services in 2020; Uber has proposed an Uber-green plan to replace traditional fuel vehicles with electric vehicles. In the following context, the vehicles operated on the platform are named as electric taxies (ETs for short). However, compared with fuel taxies, ETs have two main differences: First, ETs usually need to charge for one or two times during its operational time, due to the limited mileages in practice (200-300 km); Second, the charging time from empty to full power is about 0.5-5 hours depending on fast recharging modes, which implies that ETs have to quit from the e-platform (in off-line status) for relatively long time. The charging convenience of ETs is closely related to the number and layout of charging stations. In many cities, charging stations are insufficient comparing with the increasing number of ETs and the layout of these stations is not be well designed as well. Some stations in center or business areas are very busy and drivers have to wait for a long queue for charging their ETs, while in some remote areas, the utilizations of charging stations are usually low.

Let us consider a scenario that before the coming peak time, most ETs in a district (maybe the center of a city) may be in middle or low power. Independent drivers of these ETs decide to quit from the e-platform and go to nearby charging stations. In this situation, the service capacity (i.e., the number of available ETs) of the e-platform may decreases, which cannot satisfy the coming demands at peak time. Even worse, plenty of drivers going to the same stations may incur congestion in these stations. These drivers have to wait for a long time in the
queue, which further deteriorates the low service capacity of the e-platform at peak time. The charging operations of ETs may bring dramatic fluctuations of the service capacity of the e-platform. Therefore, the e-platform needs a centralized decision and scheme for ETs' service and charging operations considering the characteristics of ETs' charging operations and the current situation of charging stations. The illustration of the order-service and charging operations in an e-platform is presented in Figure 1.

![Figure 1: Illustration of order serve and charging operation in an e-hailing platform](image)

2 Literature review

The charging problem of electric vehicles has been widely concerned in the transportation, operation optimization, and power engineering fields. However, most of the literature focuses on charging for private electric vehicles. There are few papers studied on the charging problem for an ETs fleet of the e-hailing platform. In the following, related research will be reviewed from two aspects: private electric vehicles charging problem and ETs charging problem.

2.1 Charging of private electric vehicles

At present, most of the research focuses on the charging problem of private electric vehicles and specifically considers the following two situations: charging in the community and charging in transit.

2.1.1 The problem of charging in the community

The community charging problem is usually considered to reduce the queuing and charging cost of electric vehicles under the constraints of grid resources. For a single charging station in a community, García-Álvarez, et al. (2018) and Hernández-Arauzo, et al. (2015) reduce the total charging delay under uncertain vehicle charging time and power supply capacity constraints. To coordinate charging station workload among different regions, Flath, et al. (2014) consider charging problem from the dimensions of time and space and stimulates drivers to select appropriate charging stations to balance the inter-regional grid load and charging waiting time based on price. Besides, some literature has studied how to coordinate the private electric vehicles to charge from the perspective of the power grid (Cao, et al., 2019; Umetani, et al., 2017; Wei, et al., 2014).

2.1.2 The problem of charging in transit

In addition to the research on charging in a community, some papers consider the charging in transit problem for private electric vehicles. Usually, in this kind of problem, the current electric power of electric vehicles is insufficient to support a journey, so it is necessary to select a charging station and charge for the electric vehicle on the way. To minimize the waiting time of electric vehicles, Qin, et al., (2011) have studied the selection of charging stations under a
given driving path. Under the same problem and objective function, Gusrialdi, et al., (2017) coordinates the selection of charging stations based on the collected traffic information and charging station status information. Sweda, et al., (2017) introduces charging cost into the selection of charging stations in a road network, that is, drivers reduce the charging cost caused by charging times and charging speed based on deciding where to charge and the amount of electricity.

Some papers (Schiffer, et al., 2017; Montoya, et al., 2017; Liao, et al., 2016) consider the VRP with charging station selection. He, et al., (2014) and Cen, et al., (2018) considered the problem of minimizing the charging cost under the condition of user equilibrium in the road network. In the case of vehicles charging in a road network, Sweda, et al., (2017) proposed an effective algorithm to find the optimal path and charging strategy based on the availability of power stations. Cao, et al., (2017) proposes a communication framework for electric vehicles, which recommends the relevant information of charging stations to drivers to help them choose the appropriate path and charging station. Baum, et al., (2019) takes into account a variety of charging station types, i.e. switching station, ordinary charging station and fast-charging station, as well as minimizing travel time with optional charging capacity.

2.2 The charging problem of e-hailing platform

The problem of e-hailing platform vehicles charging has not attracted enough attention, although e-hailing platforms are becoming more and more popular. There are obvious differences between e-hailing platform charging problem and private electric vehicle charging problem. First of all, e-hailing platform vehicles are serving passengers that charge is not allowed in the serving process. Secondly, the operation of an e-hailing platform is for profit, so it is necessary to consider not only how to reduce the cost factors such as charging costs and queuing time, but also serve more passengers to obtain more revenue. In this paper, we will review the related literature from two aspects: the optimization of e-hailing platform vehicle charging decisions and the joint optimization of order service and charging assignment.

2.2.1 Charging decision optimization

Some papers studied the model and algorithm to optimize charging costs for a single network. Tian, et al., (2016) predicts the current state of each ETs by detecting the historical charging data and real-time GPS trajectory and recommends the best charging station for ETs to minimize their driving distance and waiting time. Niu, et al., (2015) considered the overall charging coordination from the perspective of the ETs fleet, to minimize the total charging cost, charge station load, and maximize the utilization rate of charging equipment. Based on Niu's research, Yang, et al., (2015) considered the charging coordination optimization of ETs fleet from the perspective of the time-space dimension.

2.2.2 Joint optimize of charging and order serving

Yang, et al., (2018) proposed a two-stage charging coordination model. In the first stage, optimal charging time is selected based on the power state of electric vehicles, the time-varying income, and the queuing situation of charging stations; then, the appropriate charging stations are selected to reduce the queuing time through the game method. Ke, et al., (2019) considered a shared online platform where ETs and gasoline vehicles coexist and the market grows over time. ETs’ drivers should manage their work and charging plan to balance the charging cost and operating income only from the time dimension. Sassi, et al., (2017) studied the problem of assigning orders for ETs and fuel vehicles when the passenger order information was completely confirmed. Besides, Hua, et al., (2019) considered an ET sharing platform to jointly
optimize the long-term charging station facility planning and real-time fleet operation (vehicle scheduling and charging decision-making) when the arrival of customers is uncertain.

Through the above review on private electric vehicles charging problem and e-hailing platform charging problem, we can find that most papers focus on the charging decision optimization of private electric vehicles in the community and transit. The optimization goal is always to minimize the charging cost of electric vehicles or the load of the community power grid. For the charging problem of the e-hailing platform, the existing parser only put forward the charging scheduling model and algorithm from the perspective of minimizing charging costs (including queuing time) of single ET. Some papers have studied the joint optimization of order service and charging assignment for the e-hailing platform but assuming that the order demand information is known. The optimization problem is established as a deterministic assignment scheduling model. Few papers can make decisions based on real-time information.

The remainder of the paper is organized as follows. Section 3 analysis the charging and order serving problem. And a multi-period stochastic model will be built. In Section 4, we convert the stochastic model into deterministic model based on the rolling-horizon framework. In Section 5, the performance of the model will discuss throughout the numerical experiment. And the conclusions are drawn in Section 6.

3 Problem analysis and formulation

In this section, we first analyze the proposed charging-and-order-serve decision (COSD) system, and then formally state the problem and corresponding assumptions.

3.1 Charging-and-order-serve decision system

The COSD system is a tactical level module embedded in the e-hailing platform, as illustrated in Figure 2. In this study, an e-hailing fleet with serval electric taxes is considered. The real-time status of the fleet can be collected to e-platform which includes availability, location, state of charge (SOC), and so on. The status of ETs and charging stations can be monitored with the support of advanced technologies, such as IoT, GPS, etc. With collected real-time information, the COSD system will make the order-serving decision and charging decision jointly to maximize the total revenue of all ETs. More specifically, the system will determine which ETs should serve coming orders and which ETs should be charged. After the decision, the system will collect the information of arrival orders and assign ETs to serve them based on real-time matching and routing module and compute a price for passengers based on the pricing module. And ETs which should be charged will drive to a central charging station. The operation of the platform is complex and is the coordination of many modules. However, in this study, we are forced on the COSD system and simplify other modules.
Dynamic Optimal Approach for an Electric Taxi Fleet's Charging and Order-service Schemes

3.2 Problem description

Let us consider an e-hailing platform that manages an electric taxi fleet to provide exclusive delivering service for passengers. The platform only operates in a finite time horizon and can be divided into \( T \) discrete periods. The period index is denoted as \( t, t \in \{1, 2, \ldots, T\} \). Under this discrete framework, at beginning of each period, the platform will make decisions based on the current state of all ETs and the workload of charging station. An ET can be one of the following statuses: 1) being available and wait to serve coming orders; 2) on served and 3) being charged at charging station. And we use \( A_t \) to represent all available ETs of fleet and \( P_t \) to represent all charging ETs at the beginning of period \( t \). Normally, we have \( |A_t| \cup |P_t| = E \), where \( E \) denotes the number of all ETs of fleet. Without loss of generality, we assume that ETs are homogeneous with the same battery capacity, and discretize into \( K \) levels. Through some IOT technologies, the platform can obtain the residual electricity of vehicles at any time. For an ET with \( k, k \in \{1, \ldots, K\} \) level, its SOC is in interval \( \left( \frac{k-1}{K}, \frac{k}{K} \right] \) and an ET with 0 level means it cannot serve any passengers and need to be charged immediately. The district in which the fleet operates is divided into \( I \) small cells that passengers always travel from one cell to another. The cell index is denoted as \( i, i \in I \). We assume that there will be a charging station to provide exclusive charging serve for ETs in each cell. The ETs in the cell \( i \) only charge at the charging station of cell \( i \). The time and SOC consumed from the location to the charging station can be ignored. Also, we assume that charging one SOC level will consume one period. Thus, the available ET set at period \( t \) is \( S_t = \{s_{t,i,k}|0 < k \leq K, i \in I\} \), where \( a_{t,i,k} \) is the number of available ETs with level \( k \) in cell \( i \). And the out-serve ETs is denoted as \( P_t = \{p_{t,i,w}|0 < w \leq K\} \), where \( p_{t,i,w} \) is the number of ETs with type \( w \) in cell \( i \) at period \( t \).

Similarly, passenger's orders can be classified into \( K \) types according to the requested power from the origin to the destination that passengers released. The order type index is denoted as \( l \) and computed by \( l(i, j), i, j \in I \). In this study, the passengers are randomly arriving with unknown distribution. We use \( F(\tilde{d}_{t,i,j}) = \{\tilde{d}_{t,i,j}|i, j \in I\} \) to represent the set of unknown probability distribution function, where \( d_{t,i,j} \) represent the number of orders from cell \( i \) to cell \( j \) during period \( t \), which are random variables. Besides, this study assumes that the order serve-time (i.e., ET traveling time) is an input parameter \( \vartheta(i, j) \), depending on its origin and destination, which are estimated by the advanced real-time transportation information and navigation systems, such as Google map and Amap.

Figure 2: Illustration of an e-hailing platform with COSD system
As aforementioned, the operational framework of COSD system is illustrated in Figure 3. At the beginning of period $t$, COSD system needs to make the charging decision $X_t = \{x_{t,i,k}|0 \leq k < K, i \in I\}$ and the order-serve decision $Y_t = \{y_{t,i,j,k}|0 \leq k \leq K, i, j \in I\}$. Specifically, $x_{t,i,k}$ represents the number of available ETs (i.e., $S_{t,i}$) in cell $i$ with type $k$ which need to be fully charged, and $y_{t,i,j,k}$ specifies the number of available ETs with type $k$ to serve the special orders from cell $i$ to cell $j$ which are arrival during the current period. Note that an order with type $l(i,j)$ can only be served by type $k$ ETs, which satisfy $k \geq l(i,j)$. During the period, some orders may be not served, thus we use $m_{t,i,j,k}$ to represent the number of orders travel cell $i$ to cell $j$ which are actually served by type $k$ ETs. If the ETs driver served orders, they will obtain a reward $r(l(i,j))$. The reward has already been subtracted from the operation cost but not the charging fee. If there are no available ETs for serving order during period $t$, the order will lose and platform will suffer a penalty cost $\theta(l(i,j))$.

Figure 3: Operation framework of COSD system

To sum up, the problem addressed in this paper is to determine the ET charging and order serving decision in a finite horizon to maximize the total reward of the fleet. Notations frequently used throughout the paper are listed in Table 1. And the problem has the following important assumptions:

- An ET assigned to an order will pick up a passenger(s) immediately and travel to another cell. The SOC and time consuming from ETs location to passengers origin is ignored
- Each cell will have a charging station with $N$ chargers. The travel time from the location of ETs to the station can be ignored. Furthermore, the station provides exclusively charging service for the ET fleet
- ET’s drivers are employees of e-hailing platform and always comply with the instructions sent from the platform. In the numerical experiment section, a decentralized setting is addressed in which drivers are self-employees and want to maximize their revenues independently.
- Lastly, we consider impatient passengers. Passengers will switch to other transportation modes, such as subways, buses, or other e-hailing platforms if their orders are not confirmed in the current period.
Table 1: main notions in this paper

<table>
<thead>
<tr>
<th>Indexes and parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Period index, $1 \leq t \leq T$</td>
</tr>
<tr>
<td>$k$</td>
<td>ET type index, $0 \leq k \leq K$</td>
</tr>
<tr>
<td>$l$</td>
<td>Set of cells</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Set of available ETs of period $t$</td>
</tr>
<tr>
<td>$s_{t,i,k}$</td>
<td>Number of available ETs of type $k$ in cell $i$ of period $t$</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Set of charging ETs of period $t$</td>
</tr>
<tr>
<td>$p_{t,i,k}$</td>
<td>Number of ETs with type $k$ in cell $i$ of period $t$</td>
</tr>
<tr>
<td>$F(\tilde{d}_{t,i,j})$</td>
<td>Set of unknown probability distribution function of $\tilde{d}_{t,i,j}$</td>
</tr>
<tr>
<td>$\tilde{d}_{t,i,j}$</td>
<td>Number of random orders from cell $i$ to cell $j$ of period $t$, $i,j \in I$</td>
</tr>
<tr>
<td>$l(i,j)$</td>
<td>Type of orders from cell $i$ to cell $j$</td>
</tr>
<tr>
<td>$r(l(i,j))$</td>
<td>Reward of serving an order with type $l(i,j)$</td>
</tr>
<tr>
<td>$\theta(l(i,j))$</td>
<td>Penalty of losing an order with type $l(i,j)$</td>
</tr>
<tr>
<td>$\vartheta(i,j)$</td>
<td>Consumed time from cell $i$ to cell $j$</td>
</tr>
<tr>
<td>$m_{t,i,j,k}$</td>
<td>Number of type $k$ ETs which are actually served order from cell $i$ to cell $j$</td>
</tr>
</tbody>
</table>

Decision variable

<table>
<thead>
<tr>
<th>$x_{t,i,k}$</th>
<th>Number of ETs should be charged in cell $i$ of period $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t,i,j,k}$</td>
<td>Number of ETs should serve coming order from cell $i$ to cell $j$ of period $t$</td>
</tr>
</tbody>
</table>

In this problem, ETs can be seen as a special kind of reusable resources. As illustrated in Figure 4, an ET with type $k$ at period $t$ will be unavailable until the beginning of period $t + K - k$, if it is assigned to be charged (dubbed as Case (a)). Or it will be available at the beginning of period $t + \vartheta(i,j)$, if it is assigned to serve order $l(i,j)$ (dubbed as Case (b)). However, different from the common reusable resource, such as equipment or machines, the ET's SOC changes when it is available once more, and its usability may thus be affected considerably. For example, the SOC is increased to $K$ in Case (a), but it decreased to $k - l(i,j)$ in Case (b). On the other hand, as the orders randomly arrive during each period, the corresponding number of assigned ETs successfully serving orders (i.e., $m_{t,i,j,k}$) is thus a random variable affected by the random orders. In this paper, we are forced on the joint scheme of ETs charging and order serving. The order arrival sequence and the assigned rule are simplified.
3.3 Problem formulations

In this section, the problem will be formulated in a multi-period stochastic model.

3.3.1 Objective function

The objective of the model is to maximize the total reward of the whole fleet over $T$ horizon. Thus, the expected reward of period $t$ is

$$ r_t^e = \sum_{i \in I} \sum_{j \in I} \sum_{l(i,j)=0}^{d(t,i,j)} [r(l(i,j)) \times \sum_{k=l(i,j)}^{K} m_{t,i,j,k} - \theta(l(i,j)) \times (d_{t,i,j} - \sum_{k=l(i,j)}^{K} m_{t,i,j,k})^+] f_t(d_{t,i,j}) $$

where $r(l(i,j)) \times \sum_{k=l(i,j)}^{K} m_{t,i,j,k}$ are the rewards of serving orders and $\theta(l(i,j)) \times (d_{t,i,j} - \sum_{k=l(i,j)}^{K} m_{t,i,j,k})^+$ is the penalty cost of the unsatisfied orders.

3.3.2 Constraints

Based on the problem statement in Section 3.2, four classes of constraints need to be considered in the problem.

1. Capacity constraints of available ETs

   Naturally, the ETs which assign to be charged and serve coming orders should less than the available ETs of different types, as shown in below constraints. Note that the ETs with full SOC don’t need to charge and ETs with zero SOC cannot serve orders.

   $$ \sum_{k=1}^{K} y_{t,i,j,k} \leq s_{t,i,K}, \quad 1 \leq t \leq T, i \in I $$

   $$ x_{t,i,k} + \sum_{j \in I} y_{t,i,j,k} \leq s_{t,i,k}, \quad 1 \leq t \leq T, 1 \leq k < K, i \in I $$

   $$ x_{t,i,0} \leq s_{t,i,0}, \quad 1 \leq t \leq T, i \in I $$

2. Served orders constraints

   In our framework, the decision is made at the beginning of each period according to the expected order quantity before the order arrival. It likely happens that some passenger maybe not served if the real number of orders more than the planned ETs quantity, i.e., $d_{t,i,j} > \sum_{k=l(i,j)}^{K} y_{t,i,j,k}$, where $d_{t,i,j}$ is the actual order quantity from cell $i$ to cell $j$. Thus, we have the following constraints:

   $$ \sum_{j \in I} m_{t,i,j,k} \leq \tilde{d}_{t,i,j}, \quad 1 \leq t \leq T, 1 \leq l(i,j) \leq K, i \in I $$

   $$ m_{t,i,j,k} \leq y_{t,i,j,k}, \quad 1 \leq t \leq T, 1 \leq l(i,j) \leq k \leq K, i \in I $$

---

Konze Yu, Pengyu Yan and Zhibin Chen
Capacity constraints of charging station

We assume that the charging station is in the center of the cell and have $N$ chargers. Based on the IoT technology, the platform can monitor the workload of charging station. Thus, the ETS assign to be charged will less than the available chargers at period $t$.

$$\sum_{k=1}^{K-1} x_{t,i,k} \leq N - \sum_{w=1}^{K-1} p_{t,i,w}, \quad 1 \leq t \leq T, i \in I$$

State transition equations

First, we consider the state transition of charging station. The number of ETs with type $w$ in cell $i$ at period $t+1$ is the number of ETs with type $w-1$ in cell $i$ at period $t$ add the number of ETs should be charged with type $w-1$ in cell $i$ at period $t$.

$$p_{t+1,i,w} = p_{t,i,w-1} + x_{t,i,w-1}, \quad 1 \leq w \leq K$$

For state transition of available ETs. The number of available ETs with type $k$ in cell $i$ at the beginning of period $t+1$ is the number of available ETs with type $k$ in cell $i$ at period $t$ minus ETs assign to serve ET with type $k$ from cell $i$ minus the number of ETs should be charged add number of ETs with type $k$ from other cells. Also, for the number of type $K$ ETs of the next period, it adds the number of ETs fully recharged at the charging station and becomes available again.

$$s_{t+1,i,k} = s_{t,i,k} - \sum_{j \in I} m_{t,i,j,k} + p_{t,k}$$

$$s_{t+1,i,k} = s_{t,i,k} - x_{t,i,k} - \sum_{j \in I} m_{t,i,j,k} + \sum_{j \in I} m_{(t-\partial(i,j)-1)^+,j,i,l(i,j)+k}^*, \quad 1 \leq k \leq K-1$$

$$s_{t+1,i,k} = s_{t,i,0} - x_{t,i,0} + \sum_{j \in I} m_{(t-\partial(i,j)-1)^+,j,i,l(i,j)+k}^*$$

To sum up, the multi-period stochastic model is formally presented as follows.

Problem $P_s$;

$$\max \mathbb{E}R^* = \sum_{t=1}^{T} r_t^s$$

$$s.t. (2) - (11)$$

4 Rolling-Horizon framework

In this section, a rolling-horizon framework will be applied to solve the multi-period stochastic problem. In actually, the probability distribution function of orders demand is difficult to estimate accurately. However, the historical data of orders can be easily obtained. Thus, we will first convert the stochastic model into a deterministic formulation based on the predicted means of order demand. And then, apply the rolling-horizon framework to solve the deterministic model with progressively issued orders.

4.1 Deterministic model
With historical data, the mean of orders can be easily computed and can be used to replace the random order demand variable. We use $\bar{d}_{t,i,j}$ to represent the mean of orders from cell $i$ to cell $j$ at period $t$ and have the following expect reward and constrains:

$$r_t^d = \sum_{i \in I} \sum_{j \in I} \left[ r(l(i,j)) \times \sum_{k=l(i,j)}^{K} m_{t,i,j,k} - \theta(l(i,j)) \times (\bar{d}_{t,i,j} - \sum_{k=l(i,j)}^{K} m_{t,i,j,k})^+ \right]$$ (12)

$$\sum_{j \in I} m_{t,i,j,k} \leq \bar{d}_{t,i,j}, \quad 1 \leq t \leq T, \quad 1 \leq l(i,j) \leq K, \quad i \in I$$ (13)

Other constraints don’t contain order random variables and are consistent with the corresponding constraints in the stochastic model. Thus, the deterministic model is built as:

**Problem $P_d$:**

$$\max \mathbb{E} R^* = \sum_{t=1}^{T} r_t^d$$

s. t. (2) – (4), (6) – (11), (13)

There are a variety of deterministic optimization techniques that can be directly used to solve the above problem $P_d$. In this paper, we adopt the state-of-art commercial optimization tool to solve the model. The solution quality and computational efficiency of the model are evaluated in Section 5.

### 4.2 Rolling-horizon algorithm

The rolling-horizon framework is widely used in multi-period decision problems and shows good performance. In this study, the order demands are released over the period. Once it is released, it will have no impact on subsequent decisions. The framework of rolling-horizon is shown in figure 5. The model will make the decision based on the orders mean of each following period. Before the next period, the number of arrival orders is released. The reward of current period can be computed by:

$$r_t = \sum_{i \in I} \sum_{j \in I} \left[ r(l(i,j)) \times \sum_{k=l(i,j)}^{K} m_{t,i,j,k} - \theta(l(i,j)) \times (d_{t,i,j} - \sum_{k=l(i,j)}^{K} m_{t,i,j,k})^+ \right]$$

Where $d_{t,i,j}$ is released the number of orders from cell $i$ to cell $j$ at period $t$. After that, the assignment decisions are implemented and the model will only be solved based on the order means of the following periods.
Dynamic Optimal Approach for an Electric Taxi Fleet’s Charging and Order-service Schemes

Figure 5: The rolling-horizon framework

To sum up, the algorithm of the rolling-horizon algorithm is shown below. At the beginning of the period \( t \), the platform will collect the real-time information of available ETs \( S_{t,i,k}, 0 \leq k \leq K, i \in I \) and the workload of charging station \( p_{t,i,k}, 0 \leq k \leq K, i \in I \). Based on this information and order means \( \tilde{d}_{t,i,j}, t \leq t' \leq T \), the model will be solved. After that, the charging and order-serving decision of the current period will execute to obtain actually served order \( m_{t,i,j,k} \) and revenue \( r_t \) based on the solution.

For period \( t = 1 \) to \( T \)

Step 1: Platform collects real-time information: \( S_{t,i,k} \) and \( p_{t,i,k} \) at the beginning of period \( t \);  
Step 2: Solve model \( P_d \) with the mean \( \tilde{d}_{t,i,j} \) for coming periods \( t' \), \( t \leq t' \leq T \);  
Step 3: Execute the charging and order-service assignments according to solution \( x_{t,i,k} \) and \( y_{t,i,j,k} \);  
Step 4: At the end of period \( t \), the served demands \( m_{t,i,j,k} \) are realized and the platform obtains the corresponding revenue \( r_t \).

End for

5 Numerical experiment

In this section, the performance of the model will be discussed. The historical order data were collected from Didi’s GAIA open dataset in the center area of Chengdu City from November 1 to November 30, 2016. The region of order is divided into 9 cells, which is shown in figure 6. In the experiment, the operation horizon is from 6 a.m. to 12 p.m. the period length is set as 15 minutes. The parameters setting is shown in table 2. The deterministic model is solved by IBM ILOG CPLEX Optimization Studio V12.8.0 using a PC with i7 CPU @ 3.00GHz and 8.00G RAM.
Figure 6: Illustration of regional division and charging station location.

Table 2: numerical experiment parameters setting

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>108 period</td>
</tr>
<tr>
<td>$K$</td>
<td>10 level</td>
</tr>
<tr>
<td>$I$</td>
<td>9 cells</td>
</tr>
<tr>
<td>$N$</td>
<td>20 chargers</td>
</tr>
<tr>
<td>$E$</td>
<td>100 ETs</td>
</tr>
</tbody>
</table>

First, we will discuss the performance of the model compared with the post-optimal solution and decentralized case. In the post-optimal solution, the model is solved based on all released order demands and we use $Gap2 = \frac{R^*-R^d}{R^*}$ to represent the solution gap between our model and the post-optimal model, where $R^*$ is the solution of post-optimal model and $R^*$ is the solution of our model. In the decentralized case, all drivers make charging decisions that are based on their knowledge and only go to enter the charging station where they are located. In this situation, it will happen that many ETs wait at the same charging station and waste their operation time. The arrival orders are randomly assigned the available ETs with the constrain $k \geq l(i, j)$. And the $Gap1 = \frac{R^*-R^{dec}}{R^*}$ to represent the solution gap between the decentralized solution and post-optimal solution, where $R^{dec}$ is the solution to decentralized strategy. And the experiment simulates 20 weekdays based on Didi’s historical data. The result is shown in figure 7. It can be seen that our solution is far better than the solution of decentralized. The average gap of $Gap1$ is 232% and the average $Gap2$ is 47.1%.
Then, we compare the $\text{Gap}_1$ and $\text{Gap}_2$ in different chargers. The result is shown in figure 8. It can be seen that our model is better than the decentralized decision in different chargers. And $\text{Gap}_1$ and $\text{Gap}_2$ will decrease with the number of chargers increase.

### Figure 8: The changes of Gap1 and Gap2 under different chargers

#### 6 Conclusion

In this paper, a charging and order serving problem is described. Compared with flue taxi, the milage of ETs are relatively short so that ETs’ driver always recharges the battery during operation time which will waste their time, impact the platform serving capacity, and reduce their revenue. Thus, we propose a centralized COSD system to solve this problem. with the current information of available ET and charging station workload, the decision will be made based on a multi-period stochastic model. To solve the model, we convert it into a deterministic model under the rolling-horizon framework. During the numerical experiment, our model is far better than the decentralized strategy and shown a better performance with chargers increase.

In future work, we will use the data-driven approach like SAA to replace the order means when converting the stochastic model into the deterministic. Also, the model can be built in the framework of the Markov Stochastic Process and use some state-of-art approaches like reinforcement learning to solve the problem. In this paper, we assume that drivers are unconditionally subject to decisions made by the platform. However, in practice, drivers have
their charging preferences. Therefore, in the following study, we will focus on how to set incentive prices to make drivers willing to obey the decisions made by the COSD system.

References

- Qin Hua, Zhang Wensheng, (2011) : Charging scheduling with minimal waiting in a network of electric vehicles and charging stations; proceedings of the Proceedings of the Eighth ACM international workshop on Vehicular inter-networking, F, ACM.
Dynamic Optimal Approach for an Electric Taxi Fleet's Charging and Order-service Schemes